

Theoretical Analysis of Perturbed Geostationary Communication Satellite Orbit

Abdulkadir Iyyaka Audu, Jibril Danladi Jiya

Abstract— Development and orbital analysis of nonlinear model of geostationary communication satellite is presented. Satellite stability in orbit is the most challenging aspect of satellite technology. Satellites generally have the required instrumentation to provide intercontinental communication services. The orbital aspect of geostationary satellite is critical since it directly affects the link characteristics and has a significant impact on satellite network design. The fact that many subtle effects perturb earth satellite orbits, invalidating the simple orbits predicted by two-body gravity equations has necessitate the need to study the orbital aspect of satellite communication. This work describes a set of second-order differential equations which constitute the nonlinear model of the satellite. The model is confirmed to be sufficiently representative of the satellite since it provides the potential for very intensive verification and understanding of Kepler's laws of planetary motion that are also applicable to the motion of satellites around earth.

Keywords— Geostationary satellite, Linear model, Perturbed orbit, equatorial orbit, Keplerian law, Eccentricity, Anomaly.

1 INTRODUCTION

FAIRLY recently, communications satellites have become an attractive carriers of long distance communications.

The satellite industry has grown until it handles most international telephone traffic, all international and almost all domestic long-distance television program distribution, and a rapidly growing proportion of new domestic voice and data channels [1], [2].

Satellite communication is one of the most impressive spinoffs from the space programs and has made a major contribution to the pattern of international communications. A communication satellite is basically an electronic communication package placed in orbit whose prime objective is to initiate or assist communication transmission of information or message from one point to another through space [3].

It all began with an article by Arthur C. Clarke published in October 1945 issue of *Wireless World*, which theoretically proposed the feasibility of establishing a communication satellite in a geostationary orbit. In that article, he discussed how a geostationary orbit satellite would look static to an observer on Earth within the satellite's coverage, thus providing an uninterrupted communication service across the globe. This marked the beginning of the satellite era [4].

Due to the nature of satellite links (long propagation delay, relative high bit error rate and limited bandwidth in comparison with terrestrial links, particularly optical links), some standard network protocols do not perform well and have to be adapted to support efficient connection over satellite. Satellite orbit directly affects the link characteristics [5] and has a significant impact in satellite network design [6].

A satellite in a circular orbit at such an altitude that corresponds to an orbital period of one day revolves around earth at the same speed as earth's rotation. This altitude is 35,786.6 km, and the orbit is called a synchronous or geosynchronous orbit. If inclination of a

geosynchronous orbit is zero (or near zero), the satellite remains fixed (or approximately fixed) over one point on the equator. Such an orbit is known as a geostationary orbit. An advantage of the geostationary orbit is that antennas on the ground, once aimed at the satellite, need not continue to rotate. Another advantage is that a satellite in this type of orbit continuously sees about one-third of earth. At an altitude of 35,786.6 km above the equator, the angular velocity of a satellite in this orbit matches the daily rotation of the earth's surface, and this orbit has been widely used as a result [7]. It is well known that a system of three satellites in geostationary orbit each separated by 120 degrees of longitude, as shown in Fig. 1, can receive and send radio signals over almost all the inhabited portions of the globe [8]

Many subtle effects such as earth's oblateness, solar and lunar effects, solar radiation pressure perturb earth satellite orbits, invalidating the simple orbits predicted by two-body gravity equations [9], [10]. Therefore, a study of nonlinear model of geostationary communication satellite will provide an insight to the orbital parameters which ensure stability under these subtle effects.

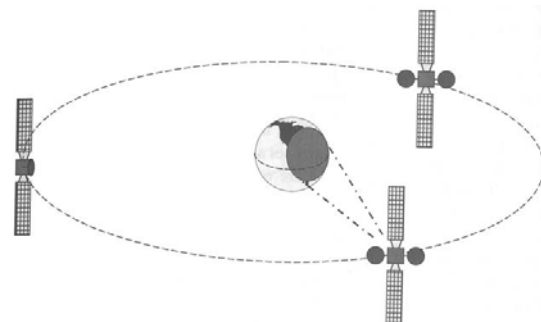


Fig. 1 A system of three geostationary communication satellites provides nearly worldwide coverage.

This paper is organized as follows. The introduction to this work is presented in section 1. Satellite model description is given in section 2. Section III is devoted to model analysis. Discussion is draw concerning the focus of the paper in section 4.

2 GEOSTATIONARY SATELLITE MODEL

From Newton's law of universal gravitation, the gravitational force, \vec{F}_o on a satellite of mass m located at a vector distance r from the centre of the earth can be expressed as

$$\vec{F} = -\frac{GM_E m \hat{r}}{r^2} \tag{1}$$

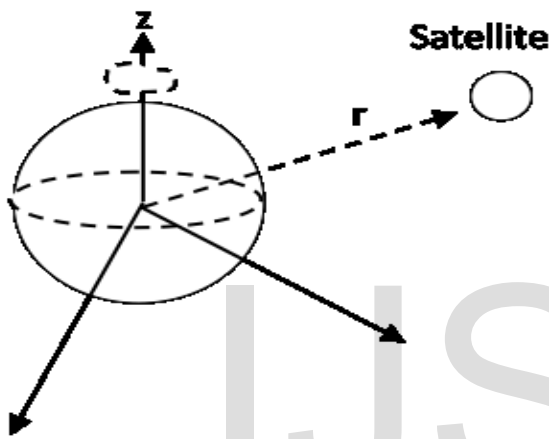


Fig. 2 This rectangular coordinate system describes the earth and a geostationary satellite. The x, y, and z axes are fixed by earth's equatorial plane and north-pole. The satellite is located with vector with respect to the centre of the earth.

In (1), M_E is the mass of the earth, G is the gravitational constant \hat{r} , is a unit vector in r_o , direction. For the same satellite, Newton's second law of motion is can be written as

$$\vec{F} = m \frac{d^2 r}{dt^2} \hat{r} \tag{2}$$

Equating (1) and (2), and let $GM_E = \mu$. GM_E is called Kepler's constant , we have

$$\frac{d^2 r}{dt^2} \hat{r} + \frac{\mu \hat{r}}{r^2} = 0 \tag{3}$$

But

$$\hat{r} = \frac{\vec{r}}{r} \tag{4}$$

Therefore, (3) becomes

$$\frac{d^2 \vec{r}}{dt^2} + \frac{\mu \vec{r}}{r^3} = 0 \tag{5}$$

Equation (5) can be solved by expressing it in a second rectangular coordinate system where the unit vectors are constant. A second rectangular coordinate system (x_o, y_o, z_o) defined by orbital plane possible such that we can write

$$\vec{r} = x_o \hat{X} + y_o \hat{Y} \tag{6}$$

Substituting (6) in (5), we get

$$\hat{X}_o \left(\frac{d^2 x_o}{dt^2} \right) + \hat{Y}_o \left(\frac{d^2 y_o}{dt^2} \right) + \frac{\mu (x_o \hat{X}_o + y_o \hat{Y}_o)}{(x_o^2 + y_o^2)^{3/2}} = 0 \tag{7}$$

The following transformations could be used to express (7) in polar coordinate and consequently lead to its simplification.

$$x_o = r_o \cos \beta_o \tag{8.1}$$

$$y_o = r_o \sin \beta_o \tag{8.2}$$

$$\hat{X}_o = \hat{r}_o \cos \beta_o - \hat{\beta}_o \sin \beta_o \tag{8.3}$$

$$\hat{Y}_o = \hat{\beta}_o \cos \beta_o + \hat{r}_o \sin \beta_o \tag{8.4}$$

From (8.1) we can write

$$\frac{dx_o}{dt} = -r_o \sin \beta_o \frac{d\beta_o}{dt} + \cos \beta_o \frac{dr_o}{dt}$$

$$\begin{aligned} \frac{d^2 x_o}{dt^2} = & -r_o \sin \beta_o \left(\frac{d^2 \beta_o}{dt^2} \right) - r_o \cos \beta_o \left(\frac{d\beta_o}{dt} \right)^2 - 2 \sin \beta_o \left(\frac{d\beta_o}{dt} \right) \left(\frac{dr_o}{dt} \right) + \\ & \cos \beta_o \left(\frac{d^2 r_o}{dt^2} \right) \end{aligned} \tag{9}$$

Also,

$$\frac{dy_o}{dt} = r_o \cos \beta_o \frac{d\beta_o}{dt} + \sin \beta_o \frac{dr_o}{dt}$$

$$\begin{aligned} \frac{d^2 y_o}{dt^2} = & r_o \cos \beta_o \left(\frac{d^2 \beta_o}{dt^2} \right) - r_o \sin \beta_o \left(\frac{d\beta_o}{dt} \right)^2 + 2 \cos \beta_o \left(\frac{d\beta_o}{dt} \right) \left(\frac{dr_o}{dt} \right) + \\ & \sin \beta_o \left(\frac{d^2 r_o}{dt^2} \right) \end{aligned} \tag{10}$$

Therefore,

- *Abdulkadir Iyyaka Audu* is of the Depaertment of Computer Engineering, Faculty of Engineering in University of Maiduguri, Nigeria, Mobile no.- +234(0)7097252528. E-mail: lm324fairchild@gmail.com
- *Jibril Danladi Jiya* lectures in the Department of Mechatronc Engineering, Abubakar Tafawa Balewa University Bauchi, Nigeria, Mobile no.- +234(0)7032153315. E-mail: jibjiya@yahoo.co.uk

$$\widehat{X}_o \left(\frac{d^2 x_o}{dt^2} \right) = [\widehat{r}_o \cos \beta_o - \widehat{\beta}_o \sin \beta_o] \left[-r_o \sin \beta_o \left(\frac{d^2 \beta_o}{dt^2} \right) - r_o \cos \beta_o \dot{\beta}_o \dot{\beta}_o dt - 2 \sin \beta_o \dot{\beta}_o \dot{\beta}_o dt + \cos \beta_o 2 \dot{\beta}_o \dot{\beta}_o dt \right] \quad (11)$$

$$\widehat{Y}_o \left(\frac{d^2 y_o}{dt^2} \right) = [\widehat{\beta}_o \cos \beta_o + \widehat{r}_o \sin \beta_o] \left[r_o \cos \beta_o \frac{d^2 \beta_o}{dt^2} - r_o \sin \beta_o \dot{\beta}_o \dot{\beta}_o dt + 2 \cos \beta_o \dot{\beta}_o \dot{\beta}_o dt + \sin \beta_o 2 \dot{\beta}_o \dot{\beta}_o dt \right] \quad (12)$$

$$\frac{\mu(x_o \widehat{X}_o + y_o \widehat{Y}_o)}{(x_o^2 + y_o^2)^{3/2}} = \frac{\mu([r_o \cos \beta_o][\widehat{r}_o \cos \beta_o - \widehat{\beta}_o \sin \beta_o] + [r_o \sin \beta_o][\widehat{\beta}_o \cos \beta_o + \widehat{r}_o \sin \beta_o])}{(r_o^2 (\cos \beta_o)^2 + r_o^2 (\sin \beta_o)^2)^{3/2}} = \frac{\mu}{r_o^2} \quad (13)$$

Considering \widehat{r}_o and $\widehat{\beta}_o$ components of (7) we can write

$$\frac{d^2 r_o}{dt^2} - r_o \left(\frac{d\beta_o}{dt} \right)^2 = -\frac{\mu}{r_o^2} \quad (14)$$

$$r_o \left(\frac{d^2 \beta_o}{dt^2} \right) + 2 \left(\frac{dr_o}{dt} \right) \left(\frac{d\beta_o}{dt} \right) = 0 \quad (15)$$

Equation (14) and (15) constitute a pair of second order differential equations which describe the motion of a satellite. One disadvantage of the geostationary orbit is that the gravity of the sun and moon disturb the orbit, causing the orbital inclination to increase. The satellite's propulsion can counter this disturbance. Therefore, we have to factor the propulsion in both r_o and β_o directions in (14) and (15) respectively. Hence, we have

$$\frac{d^2 r_o}{dt^2} - r_o \left(\frac{d\beta_o}{dt} \right)^2 - \frac{f_{r_o}}{m} = -\frac{\mu}{r_o^2} \quad (16)$$

$$r_o \left(\frac{d^2 \beta_o}{dt^2} \right) + 2 \left(\frac{dr_o}{dt} \right) \left(\frac{d\beta_o}{dt} \right) - \frac{f_{\beta_o}}{m r_o} = 0 \quad (17)$$

3 MODEL ANALYSIS AND DISCUSSION

3.1 Orbit Description Analysis

Since m , is large compared to f_{β_o} , (17) is approximately equals to (15). To obtain the equation (i.e orbital equation) which relates r_o to the angle of sweep β_o , of the orbit plane, (16) has to be solved. Equation (15) is rearranged as follows

$$\frac{1}{r_o} \frac{d}{dt} \left(r_o^2 \frac{d\beta_o}{dt} \right) = 0 \quad (18)$$

Equation (18) is only true if $r_o^2 \frac{d\beta_o}{dt}$ is a constant, hence

$$r_o^2 \frac{d\beta_o}{dt} = \text{a constant} = g \quad (19)$$

Where g is the magnitude of a vector called angular momentum. Squaring both sides of (19) and rearranging, it becomes

$$r_o \left(\frac{d\beta_o}{dt} \right)^2 = \frac{g^2}{r_o^3} \quad (20)$$

Substituting for $r_o \left(\frac{d\beta_o}{dt} \right)^2$ in (16) we have

$$\frac{d^2 r_o}{dt^2} - \frac{g^2}{r_o^3} - \frac{f_{r_o}}{m} = -\frac{\mu}{r_o^2} \quad (21)$$

To obtain the angular (β_o) rate of change of r_o , we make use of function of a function theorem. Let

$$D = \frac{1}{r_o} \quad (22)$$

Therefore,

$$\frac{dr_o}{d\beta_o} = -\frac{1}{D^2} \frac{dD}{d\beta_o} \quad (23)$$

But

$$\frac{dr_o}{dt} = \left(\frac{dr_o}{d\beta_o} \right) \left(\frac{d\beta_o}{dt} \right) = \left(-\frac{1}{D^2} \frac{dD}{d\beta_o} \right) \left(\frac{g}{r_o^2} \right) = -g \frac{dD}{\beta_o} \quad (24)$$

Therefore,

$$\frac{d^2 r_o}{dt^2} = -g^2 D^2 \left(\frac{d^2 D}{d\beta_o^2} \right) \quad (25)$$

Equation (21) becomes

$$-g^2 D^2 \left(\frac{d^2 D}{d\beta_o^2} \right) - \frac{g^2}{r_o^3} - \frac{f_{r_o}}{m} = -\frac{\mu}{r_o^2} \quad (26)$$

Since m is large compare to f_{r_o} (26) reduces to

$$-g^2 D^2 \left(\frac{d^2 D}{d\beta_o^2} \right) - \frac{g^2}{r_o^3} = -\frac{\mu}{r_o^2} \quad (27)$$

$$\left(\frac{d^2 D}{d\beta_o^2} \right) - D = -\frac{\mu}{g^2}$$

The solution of the second-order differential equation expressed in (27) is

$$D = \frac{\mu}{g^2} + K \cos(\beta_o - \theta)$$

Consequently,

$$r_o = \frac{p}{1 + e \cos(\beta_o - \theta)} \quad (28)$$

Where $\frac{g^2 K}{\mu} = e$, the eccentricity of the orbit, $p = \frac{g^2}{\mu}$, K and θ are constants to be determined based on certain boundary conditions. When $\theta = 0$, i.e the orbital plane coincides with the equatorial plane,

$$r_o = \frac{p}{1 + e \cos \beta_o} \quad (29)$$

The linear model of a perturbed geostationary satellite has the potential for very intensive verification of satellite's Keplerian law of motion. Equation (29) confirms that the orbit of a geostationary satellite lies in a plane and is an ellipse in which one focus is the earth. The motion of a geostationary satellite is illustrated in Fig. 3 [6].

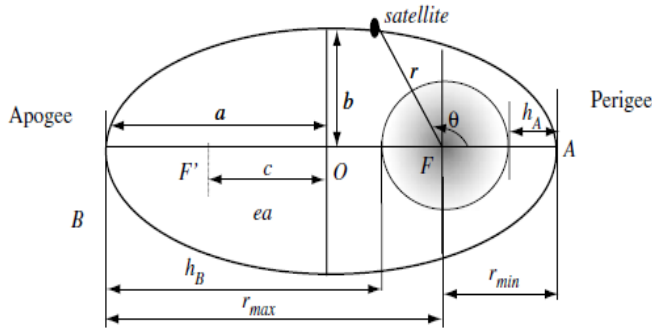


Fig. 3 Geostationary satellite orbit.

In Fig. 3, the point A of closest approach to the earth is called the perigee and the farthest B is called the apogee. The centre of the earth coincides with one of the foci F. The label a is the semi-major axis and b is the semi-minor axis.

Consider (19) i.e

$$r_o^2 \frac{d\beta_o}{dt} = g \tag{30}$$

$$\frac{1}{2} r_o^2 \frac{d\beta_o}{dt} = \frac{1}{2} g = \frac{1}{2} \sqrt{\mu p} \tag{31}$$

The right hand side of (31) equals the differential area of solid angle $d\beta_o$. Therefore, (31) confirms that the orbit of the satellite sweeps out equal areas in equal time. This is a confirmation of Kepler's second law of planetary motion. From Fig. 3, p is given as

$$p = a(1 - e^2) \tag{32}$$

The area of the ellipse is given as

$$\text{Area} = \pi ab \tag{33}$$

The orbital period T of the ellipse can be obtained by equating the right hand side of (3) and the swept out in one orbital revolution, i.e

$$\pi ab = \int_0^T \frac{1}{2} \sqrt{\mu p} dt \tag{34}$$

$$T = \frac{2\pi ab}{\sqrt{\mu p}} \tag{35}$$

$$p = \frac{b^2}{a} \tag{36}$$

$$T = \frac{2\pi}{\mu^{\frac{3}{2}}} a^{\frac{3}{2}} \tag{37}$$

$$T^2 = \frac{4\pi^2}{\mu} a^3 \tag{38}$$

Equation (38) expresses Kepler's third law of planetary motion which states that the square of the period T of revolution of the smaller body about a large body is proportional to the third power of the semi major axis of the orbital ellipse.

In conclusion, therefore, (29), (31) and (38) describe the satellite orbit. When the angle of inclination is zero and T = 24 hours, then a geostationary orbit which is a specialized geosynchronous orbit is obtained.

3.2 Satellite Location Analysis

Recall (21) and assume $f_{r_o} = 0$ we have

$$\frac{d^2 r_o}{dt^2} - \frac{g^2}{r_o^3} = -\frac{\mu}{r_o^2} \tag{39}$$

Substituting for g^2 in (39), we have

$$\frac{d^2 r_o}{dt^2} - \frac{p\mu}{r_o^3} + \frac{\mu}{r_o^2} = 0 \tag{40}$$

The integration with respect to time t of (40) gives

$$\frac{1}{2} \left(\frac{dr_o}{dt} \right)^2 - \frac{a(1-e^2)\mu}{2r_o^2} - \frac{\mu}{r_o} = \int 0 dr = C \tag{41}$$

Where, C is a constant of integration. Now we have to use the boundary conditions that at perigee, $\frac{dr_o}{dt} = 0$ and $r = r_{min} = a(1 - e)$ to find the value of C. Therefore, we have C to be

$$C = \frac{a(1-e^2)\mu}{2a^2(1-e^2)} - \frac{\mu}{a(1-e)} = -\frac{\mu}{2a} \tag{42}$$

Therefore, from (41), $\left(\frac{dr_o}{dt} \right)^2$ becomes

$$\left(\frac{dr_o}{dt} \right)^2 = -2 \left[\frac{a(1-e^2)\mu}{2r_o^2} - \frac{\mu}{r_o} + \frac{\mu}{2a} \right] \tag{43}$$

From (6), the square of satellite velocity, γ as

$$\gamma^2 = \left(\frac{dx_o}{dt} \right)^2 + \left(\frac{dy_o}{dt} \right)^2 \tag{44}$$

From (8.1) we can write

$$\frac{dx_o}{dt} = -r_o \sin \beta_o \frac{d\beta_o}{dt} + \cos \beta_o \frac{dr_o}{dt} \tag{45}$$

$$\begin{aligned} \left(\frac{dx_o}{dt} \right)^2 &= r_o^2 \left(\frac{d\beta_o}{dt} \right)^2 (\sin \beta_o)^2 - 2r_o \left(\frac{d\beta_o}{dt} \right) \left(\frac{dr_o}{dt} \right) \sin \beta_o \cos \beta_o + \\ &\left(\frac{dr_o}{dt} \right)^2 (\cos \beta_o)^2 \end{aligned} \tag{46}$$

From (8.2) we can write

$$\frac{dy_o}{dt} = r_o \cos \beta_o \frac{d\beta_o}{dt} + \sin \beta_o \frac{dr_o}{dt} \quad (47)$$

$$\left(\frac{dy_o}{dt}\right)^2 = r_o^2 \left(\frac{d\beta_o}{dt}\right)^2 (\cos \beta_o)^2 + 2r_o \left(\frac{d\beta_o}{dt}\right) \left(\frac{dr_o}{dt}\right) \sin \beta_o \cos \beta_o + \left(\frac{dr_o}{dt}\right)^2 (\sin \beta_o)^2 \quad (48)$$

Therefore, (44) becomes

$$\gamma^2 = \left(\frac{dx_o}{dt}\right)^2 + \left(\frac{dy_o}{dt}\right)^2 = \left(\frac{dr_o}{dt}\right)^2 + r_o^2 \left(\frac{d\beta_o}{dt}\right)^2 \quad (49)$$

From (20), we can write

$$r_o^2 \left(\frac{d\beta_o}{dt}\right)^2 = \frac{g^2}{r_o^2} = \frac{\mu}{r_o^2} = \frac{a\mu(1-e^2)}{r_o^2} \quad (50)$$

Therefore, using (43) and (50), (49) becomes

$$\begin{aligned} \gamma^2 &= -2 \left[\frac{a(1-e^2)\mu}{2r_o^2} - \frac{\mu}{r_o} + \frac{\mu}{2a} \right] + \frac{a\mu(1-e^2)}{r_o^2} = \frac{2\mu}{r_o} - \frac{\mu}{a} \\ &= \left(\frac{\mu}{a}\right) \left(\frac{2a}{r_o} - 1\right) \end{aligned} \quad (51)$$

Equation (49) can now be written as

$$\left(\frac{\mu}{a}\right) \left(\frac{2a}{r_o} - 1\right) = \left(\frac{dr_o}{dt}\right)^2 + \frac{a\mu(1-e^2)}{r_o^2} \quad (52)$$

We can make $\left(\frac{dr_o}{dt}\right)^2$ the subject of the formula. Therefore, we get

$$\frac{dr_o}{dt} = \left[\left(\frac{\mu}{ar_o^2}\right) (a^2e^2 - [a - r_o]^2) \right]^{\frac{1}{2}} \quad (53)$$

$$dt = r_o \left(\frac{a}{\mu}\right)^{\frac{1}{2}} \left(\frac{dr_o}{[a^2e^2 - [a - r_o]^2]^{\frac{1}{2}}} \right) \quad (54)$$

The average angular velocity v of the satellite is

$$v = \frac{2\pi}{T} = \frac{1}{a} \left(\frac{\mu}{a}\right)^{\frac{1}{2}} \quad (55)$$

$$vdt = \left(\frac{r_o}{a}\right) \frac{dr_o}{[a^2e^2 - [a - r_o]^2]^{\frac{1}{2}}} \quad (56)$$

The satellite location in the orbital plane coordinate system is specified by (x_o, y_o) . A vertical line through the satellite intersects a circumscribed circle at a point such that the eccentric anomaly E is the angle from x_o axis to the line joining that point to the centre of the circumscribed circle. The mathematical relationship between E and r_o is

$$r_o = a(1 - e \cos E) \quad (57)$$

Also

$$a - r_o = ae \cos E \quad (58)$$

$$\frac{dr_o}{dE} = ae \sin E \quad (59)$$

Replacing dr_o , $(a - r_o)$, and dr_o in (52), we get

$$vdt = \left[\frac{a(1-e \cos E)}{a} \right] \frac{ae \sin E dE}{[a^2e^2 - a^2e^2(\cos E)^2]^{\frac{1}{2}}} \quad (60)$$

$$vdt = \left[\frac{a(1-e \cos E)}{a} \right] \frac{ae \sin E dE}{ae \sin E} = (1 - e \cos E) \quad (61)$$

The integration of both sides of (61) i.e

$$\int_t^{t_p} vdt = \int_0^E (1 - e \cos E) dE \quad (62)$$

Where t_p is the time when the satellite is crossing the x_o axis. This also called the time of perigee. Therefore, (61) becomes

$$v(t - t_p) = E - e \sin E \quad (63)$$

$$M = E - e \sin E \quad (64)$$

M is called the mean anomaly. Using (32) in (28) we get

$$r_o = \frac{a(1-e^2)}{1 + \cos \beta_o} \quad (65)$$

From the above analysis it is clear that (r_o, β_o) and consequently (x_o, y_o) can be calculated when e , a , and t_o are known.

3.3 Energy Analysis

The extraction algorithm process is the inverse of the embedding process. It is assumed that the watermark as well as the see value is available at the receiver end to the authorized users.

Because of atmospheric drag a satellite is expected to satisfy the law of conservation of energy [11]. Equation (24) is expressed as

$$\frac{dr_o}{dt} = g \left(-\frac{dD}{\beta_o} \hat{r}_o + D \hat{\beta}_o \right) \quad (66)$$

From (66), the square of the magnitude V of the velocity is given as

$$V^2 = g^2 \left[\left(\frac{dD}{dt}\right)^2 + D^2 \right] \quad (67)$$

Since the force between the earth and the satellite is attractive in nature, $(1 = -\epsilon)$, [12], we can rewrite (28) in terms of mechanical energy ϵ as

$$D = \frac{-\epsilon + 1 + e \cos(\beta_0 - \theta)}{p} \quad (68a)$$

$$\frac{dD}{d\beta_0} = \frac{p \frac{d}{d\beta_0} [-\epsilon + 1 + e \cos(\beta_0 - \theta)] - [-\epsilon + 1 + e \cos(\beta_0 - \theta)] \frac{d}{d\beta_0}(p)}{p^2} \quad (68b)$$

$$\frac{dD}{d\beta_0} = -\frac{e}{p} \sin(\beta_0 - \theta) \quad (69)$$

Substituting for $\left(\frac{dD}{dt}\right)^2$ and D^2 in (67), we get

$$\epsilon^2 - 2\epsilon e \cos(\beta_0 - \theta) + e^2 = \frac{p^2 V^2}{g^2} \quad (70)$$

Using

$$e \cos(\beta_0 - \theta) = Dp + \epsilon \text{ and } p = \frac{g^2}{\mu}, \text{ we get}$$

$$e^2 = \epsilon^2 + \frac{g^2}{\mu^2} \left(2\epsilon \mu \frac{1}{r_0} + V^2 \right) \quad (71)$$

Since $\mu, g,$ and e are constants, we can define another constant H such that

$$H = V^2 + 2\epsilon \frac{\mu}{r_0} \quad (72)$$

The force expressed in (1) and the potential energy U , of geostationary satellite are related [13], as follows

$$F = -\nabla U \quad (73)$$

Where, $U = -\frac{m\mu}{r_0}$. Also the kinetic energy of the satellite equals $\frac{1}{2} mV^2$. Therefore, the mechanical (total) energy ϵ of the satellite is

$$\epsilon = \frac{1}{2} m \left(V^2 - \frac{2\mu}{r_0} \right) \quad (74)$$

$$\epsilon = \frac{1}{2} m \left(\left(V^2 + 2\epsilon \frac{\mu}{r_0} \right) - \frac{2\mu}{r_0} (1 + \epsilon) \right) \quad (75)$$

Since $1 = -\epsilon$, (75) becomes

$$\epsilon = \frac{1}{2} m \left(V^2 + 2\epsilon \frac{\mu}{r_0} \right) = \frac{1}{2} mH \quad (76)$$

The fact that ϵ is constant confirms the law of conservation of mechanical energy of a satellite in geostationary orbit.

4 CONCLUSION

Development and orbital analysis of nonlinear model of perturbed geostationary communication satellite has been the focused of this paper. A satellite in geostationary orbit is expected to obey the Keplerian laws of satellite motion even when it is perturbed. Both eccentric and mean

anomalies are required in the computation of satellite position in the $x_0 - y_0$ plane.

In the study of perturbed satellite motion, the mass of the satellite is relevant in certain specific instances, such as the study of air resistance in the upper atmosphere or radiation pressure and hence the conservation of mechanical energy.

This work provides the understanding that the orbit of a geostationary satellite has to be defined and that satellite propulsion from within is required to keep it in orbit.

Modeling of the dynamical nature of the satellite system (16) and (17) provide a basis for completely dependent optimal control input (f_{r_0}, f_{β_0}) and stability analysis.

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